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Scintillation Effects on Centroid Anisoplanatism

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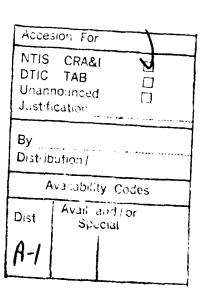
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I. INTRODUCTION

Atmospheric turbulence effects are severe for many laser systems operating in the atmosphere. Unless some sort of adaptive optics corrections are employed, these effects will greatly reduce the viability of such systems. In conventional phase-conjugate adaptive optics, a beacon signal that originates at the object of the imaging system or at the aim point of the laser transmitter system is used to provide information on the atmospherically induced phase errors associated with the desired propagation path. This instantaneous phase of the beacon signal measured by appropriate wavefront sensors is used to make the adaptive optics correction by conjugating this phase.

In a previous paper, Yura and Tavis 1 discussed in some detail the serious consequence of using a full aperture centroid tracking (i.e., Hartmann-type sensor) to measure overall wavefront tilt when another subsystem (figure sensor) measures all higher order phase distortions. The figure sensor and deformable mirror are used to remove all but tilt defined in terms of the phase, weighted over the aperture. The centroid tracker and steering mirror remove tilt as defined by the gradient of phase, averaged over the aperture. The remaining residual tilt is what leads to centroid anisoplanatism. This residual tilt leads to a large reduction of energy on target, namely Strehl ratios of the order of 0.03 for $D/r_0 = 100$.

In Ref. 1, Yura and Tavis also discussed the effect of scintillation on the centroid anisoplanatic error but did not provide numeric estimates. The purpose of this report is to provide numerical estimates of the effects of scintillation. It is shown that these are effects of secondary importance to centroid anisoplanatism.

II. SCINTILLATION EFFECTS ON CENTROID ANISOPLANATISM

We assume that the log-amplitude fluctuation χ is not zero, that it is normally distributed with a variance $\sigma_{\chi}^2 < 1$, and that the Rytov approximation is valid. 2

In Ref. 1 it was shown that the tilt variance $\langle 0_t^2 \rangle$ and the tilt-centroid correlation were independent of scintillation effects. Further it was shown that the centroid variance could be written as

$$\langle \theta_c^2 (\chi) \rangle = \langle \theta_c^2 \rangle s,$$
 (1)

where $\langle \theta_c^2 \rangle$ is the mean square centroid angle variance given by

$$\langle \theta_c^2 \rangle = \frac{\langle \theta_{co}^2 \rangle}{(1 - \epsilon^2)^2} \left[(1 + \epsilon^{11/3}) - \frac{2 \epsilon^2 \Gamma(\frac{17}{6}) \Gamma(\frac{11}{6})}{\Gamma(8/3)} {}_{2} F_{1}(\frac{1}{6}, -5/6, 2, \epsilon^2) \right]$$
 (2)

$$\langle \theta_{co}^2 \rangle = 13.39 \, (D/r_o)^{5/3} (kD)^{-2},$$
 (3)

D is the optics diameter, r_0 the lateral coherence length, and k is the wave number (= $2\pi/\lambda$ where λ is the wavelength). The quantity ϵ is the obscuration ratio, Γ is the gamma function, and ${}_2F_1$ is the hypergeometric function³.

In Eq. (1), S is given by

$$S = \frac{\int_{0}^{D} dr \ rK(r) \left[\nabla^{2}D(r) - 4\nabla^{2}B_{\chi}(r) - 8\left(\nabla^{2}B_{\varphi\chi}(r) \right)^{2} \right] exp\left[4B_{\chi}(r) \right]}{\int_{0}^{D} dr \ rK(r) \ \nabla^{2}D(r)}$$
(4)

where

$$D(r) = 6.88 (r/r_0)^{5/3}$$
 (5)

$$K(\vec{r}) = \frac{\int d^2R \ w(\vec{R} + 0.5\vec{r})W(\vec{R} - 0.5\vec{r})}{\left| \int d^2R \ W(\vec{R}) \right|^2},$$
 (6)

 $W(\overset{\star}{r})$ is the aperture function. For ϵ = 0 and a uniform aperture

$$K(x) = \frac{8}{\pi^2 p^2} \left[\cos^{-1} x - x (1 - x^2)^{1/2} \right]. \tag{7}$$

The log-amplitude B and the log-amplitude phase correlation functions may be written in terms of the corresponding structure functions.^{2,4} The explicit functional dependence of terms in Eq. (4) are given below.

We have²

$$B_{\chi}(\mathbf{r}) = \langle \chi^2 \rangle - \frac{1}{2} D_{\chi}(\mathbf{r})$$

$$B_{\phi\chi}(\mathbf{r}) = \langle \phi \chi \rangle - \frac{1}{2} D_{\chi\phi}(\mathbf{r})$$
(8)

$$D_{y}(\vec{r}) = 0.5 \left(D_{1}(r) + \text{Re } D_{2}(r)\right)$$
(9)

$$D_{\phi y}(\vec{r}) = -\operatorname{Im} D_{2}(\vec{r}) \tag{10}$$

where

$$D_{1}(\rho) = C_{1}\rho^{5/3} , \qquad (11)$$

$$C_{1} = \frac{0.033 \pi^{2} \frac{12}{5} \Gamma(\frac{1}{6})}{2^{5/3} \Gamma(\frac{11}{6})\cos\theta} k^{2} \int_{0}^{\infty} C_{n}^{2}(x) dx$$

$$D_{2}(\rho) = C_{2} \int_{0}^{\infty} C_{n}^{2}(x) x^{5/6} dx \left[{}_{1}F_{1}(-5/6, 1, \frac{ik\rho^{2}\cos\theta}{4x}) - 1 \right], \qquad (12)$$

$$c_2 = -\frac{0.033\pi^2 \frac{12}{5} \Gamma(\frac{1}{6}) k^{7/6} i^{5/6}}{\cos^{11/6} \theta}$$

$$\langle \chi^2 \rangle \simeq \frac{2^*0.041 \text{ k}^{7/6}}{\cos^{11/6} 9} \int_0^\infty C_n^2 (x) x^{5/6} dx$$
 (13)

The C_n^2 profile is plotted in Fig. 3, Ref. 4, and $_1F_1$ is the confluent hypergeometric function. We also note for convenience that the wave structure function Eq. (5) may also be written as D_1 [i.e., Eq. (11)]. To calculate the integrals appearing in Eq. (4), the gradient $(\sqrt[7]{})$ and Laplacian $(\sqrt[7]{})$ of D_1 and D_2 are needed. These expressions are given in Appendix A. To proceed further with the calculation, we use the integral expressions for the confluent hypergeometric functions for the magnitude of the argument less than 30 and its asymptotic expansion for values of the argument greater than 30. These expressions are given in Appendix B. The asymptotic expressions are valid over most of the integration range, indicated in Eqs. (4) to (12). For high altitudes and small values of r [Eq. (4)], the integral expression (B-5) was used to calculate the functional values at 1000 points, and a linear extrapolation was used between specified arguments. The expression S has been calculated, and the quantity S-1 is given in Table 1 as a function of obscuration ratio.

Table 1. Scintillation Effect on Centroid Variance

S-1	ϵ (obscuration ratio)	
7 × 10 ⁻⁵	0.0	
7.1×10^{-6}	0.1	
7.4×10^{-5}	0.2	
7.9×10^{-5}	0.3	
8.7×10^{-5}	0.4	
9.9×10^{-5}	0.5	
1.19×10^{-4}	0.6	
1.53×10^{-4}	0.7	
2.26×10^{-4}	0.8	
4.571×10^{-4}	0.9	

A. STREHL RATIO

The correction factor to the Strehl ratio may be found by replacing the first factor on the right-hand side of Eqs. (34) and (41), Ref. 1, by

$$\left[1 + \frac{\langle \theta_{c \varepsilon}^2 \rangle s}{\langle \theta_{t \varepsilon}^2 \rangle}\right],$$

where S is given by Eq. (4) above. When this is done, the correction to $1.8f(\epsilon)$ (Table 2, Ref. 1) is in the 4th or 5th decimal place for the large D/r_o (~100) used in the calculations.

III. CONCLUSIONS

We have outlined the calculation of the effect of scintillation on centroid anisoplanatism. We have found that the scintillation effect is three orders of magnitude smaller than the centroid anisoplanatic effect and justifies ignoring its contribution. This does not mean that scintillation can be completely ignored in the measuring of the phase for the adaptive optics correction.

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APPENDIX A

In this appendix, the analytic expressions for the gradient of ${\rm D}_2$ and the Laplacian of ${\rm D}_1$ and ${\rm D}_2$ are given.

$$\nabla^2 D_1 = \frac{25}{9} C_1 \rho^{-1/3} \tag{A-1}$$

$$\nabla D_{2}(\rho) = C_{3} \left[\int_{0}^{\infty} C_{n}^{2}(x) x^{-1/6} dx \right] F_{1} \left(\frac{1}{6}, 2, \frac{i k \rho^{2} \cos \theta}{4x} \right) \rho,$$

$$C_{3} = \frac{i k \cos \theta}{2} C_{2} \qquad (A-2)$$

$$\nabla^2 D_2(\rho) = C_3 \int_0^{\infty} C_n^2(x) x^{-1/6} dx \left[\frac{1}{6} z_1 F_1(\frac{7}{6}, 3, z) + 2 F_1(\frac{1}{6}, 2, z) \right],$$

$$z = \frac{i k \rho^2 \cos \theta}{4x} \tag{A-3}$$

APPENDIX B

Integral and asymptotic representations of the confluent hypergeometric function are:

$$\frac{\Gamma(b-a)\Gamma(a)}{\Gamma(b)} \, {}_{1}F_{1}(a,b,z) = \int_{0}^{1} \exp(zt) \, t^{a-1} (1-t)^{b-a-1} dt$$
(B-1)

Re b > Re a >0

$$\frac{1^{F_{1}(a,b,z)}}{\Gamma(b)} \cong \frac{\exp(\pm i \pi a)z^{-a}}{\Gamma(b-a)} \left\{ \sum_{n=0}^{R-1} \frac{(a_{n})(1+a-b)_{n}}{n!} (-z)^{-n} + O(|z|^{R}) \right\}$$

$$+ \frac{\exp(z)^{a-b}}{\Gamma(a)} \left\{ \sum_{n=0}^{s-1} \frac{(b-a)_n (1-a)_n}{n!} (z)^{-n} + O(|z|^R) \right\}$$
 (B-2)

$$a_0=1$$
, $a_1=a$, $a_n=(a+n-1)\cdots(a)$

|z| large: the positive sign if
$$-\frac{1}{2}$$
 π z < \frac{3}{2} π the negative sign if $-\frac{3}{2}$ π z < -\frac{1}{2} π .

Note that the restriction on Eq. (B-1) means that $_1F_1\left(-\frac{5}{6}, 1, z\right)$ does not have an immediate integral representation. Instead, the recursion relation is used:

$$bM(a,b,z) - bM(a-1,b,z) - zM(a, b+1,z) = 0$$
 (B-3)

to find

$$_{1}F_{1}(-\frac{5}{6}, 1, z) = _{1}F_{1}(\frac{1}{6}, 1, z) - z _{1}F_{1}(\frac{1}{6}, 2, z)$$
 (B-4)

The needed integral representations are given by

$${}_{1}F_{1}(\frac{1}{6}, 1, z) = \frac{1}{\Gamma(\frac{1}{6})\Gamma(\frac{5}{6})} \int_{0}^{1} \exp(zt)t^{-5/6}(1-t)^{-1/6}dt ,$$

$${}_{1}F_{1}(\frac{1}{6}, 2, z) = \frac{1}{\Gamma(\frac{1}{6})\Gamma(\frac{11}{6})} \int_{0}^{1} \exp(zt)t^{-5/6}(1-t)^{5/6}dt ,$$

$${}_{1}F_{1}(\frac{7}{6}, 3, z) = \frac{2}{\Gamma(\frac{7}{6})\Gamma(\frac{11}{6})} \int_{0}^{1} \exp(zt)t^{1/6}(1-t)^{5/6}dt . \tag{B-5}$$

The highest order terms in the asymptotic expansion are given by

$${}_{1}F_{1}(-\frac{5}{6}, 1, z) \cong \frac{\exp(-\frac{5}{6}\pi i)z^{5/6}}{\Gamma(11/6)} [1 - (5/6)^{2}z^{-1} + 0.5*(1/6)^{2}(5/6)^{2}z^{-2}],$$

$${}_{1}F_{1}(\frac{1}{6}, 2, z) \cong \frac{\exp(\frac{1}{6}\pi i)z^{-\frac{1}{6}}}{\Gamma(\frac{11}{6})} (1 + \frac{5}{36}z^{-1} - 0.5*\frac{35}{64}z^{-2})$$

$$+ \frac{\exp(z)z^{-11/16}}{\Gamma(\frac{1}{6})},$$

$$_{1}F_{1}(\frac{7}{6}, 3, z) \approx 2 \cdot \left[\frac{\exp(\frac{7}{6}\pi i)z^{-7/6}}{\Gamma(11/6)}(1 + \frac{35}{36}z^{-1} - 0.5 * \frac{455}{64}z^{-2})\right],$$

$$+\frac{\exp(z)z^{-11/6}}{\Gamma(7/6)}$$
 \right\}. (B-6)

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